Probability of Meteoroidal Penetration

FRANCIS B SHAFFER*
Northi op Space Laboratories, Hawthorne, Calif

The past technique of calculating the flux of meteoroids capable of penetrating the skin of a spaceship is critically reviewed A new, more rigorous technique, compatible with any assumed meteoroidal flux, velocity distribution, and penetration equation, is derived using integral calculus Several values of a constant convenient for calculating the penetrating flux are calculated, assuming different values of the meteoroidal flux constants, different astronomically measured velocity distributions, and the Herrmann-Jones penetration equation The variation in the probability of penetration near the ballistic limit is shown to have negligible effect on calculated values of the penetrating flux The use of a normal, rather than a Poisson, distribution to calculate the probability of penetration is discussed

WITH the advent of space travel, the attention of engineers and astronomers has been increasingly directed towards the question of how to estimate the probability of damage to a spaceship or satellite by meteoroidal impact

A meteoroid that strikes a spaceship without penetrating may produce destructive effects from material spalled from the interior of the spaceship The mechanism of spalling is discussed by Rinehart and Pearson [1] The total mass of fragments and their velocities are lower than when penetration occurs, and a mass detached by spalling is likely to be broad perpendicular to its direction of motion and thin parallel to its motion, a shape which is poorly adapted to the penetration of structures it may subsequently encounter When penetration by a hypeı velocity pellet occuıs, in addition to loss of pressurization and damage produced by fragments of the original meteoroid and from a portion of the wall, a number of other damaging effects may occur such as high temperatures, a blinding flash of light, and an explosive decompression as the incident material is oxidized by the atmosphere within the spaceship [2] Since the effects of penetration are so much more seıious, we shall assume that only the penetrating flux of meteoroids, out of the total flux of meteoroids, needs to be considered

In the past, efforts to predict the probability of meteoroidal penetration have generally followed this procedure: 1) estimate the cumulative omnidirectional meteoroidal flux as a function of mass; 2) estimate the range of impact velocities possible, and then assume a single impact velocity for all meteoroids; 3) assume a penetration equation; 4) calculate the penetrating flux as a function of the structure to be penetrated; and 5) calculate the probability of penetration by the Poisson distribution

There are several weaknesses in the procedure just mentioned The meteoroidal flux is known to be distributed unevenly, and the cumulative flux as a function of mass depends on assumptions regarding the luminous efficiency of meteors The use of a single impact velocity to represent the impact velocities of all meteoroids is not a rigorously correct process and is likely to result in only an approximate estimate of the penetrating flux, since it fails to account for the effect of the velocity distribution and the mass distribution of meteoroids on the number of meteoroids per unit time per unit area able to penetrate a given thickness Past choices of an assumed impact velocity have apparently been made on a subjective basis rather than on an objective mathematical basis and have ranged from 15–30 km/sec The penetration equations previously used have assumed that up to a certain threshold no meteoroids penetrate, and above that threshold all meteoroids penetrate; whereas it is known that, for a given structure, the transition between no penetrations and 100% penetrations occurs gradually Finally, the use of the Poisson distribution is open to question, since its use implicitly assumes that the events (in this case, the arrival of meteoroids) are independent which may not be the case for meteoroids revolving in nearly identical orbits

Now we shall present a technique for determining more rigorously what portion of the tool meteoroidal flux constitutes the penetrating flux For this purpose, some knowledge of the phenomena of hypervelocity impact is needed

There have been several equations proposed for predicting hypervelocity penetration into a semi-infinite plate:

$$p_\infty = 2\ 83 \left(\frac{\rho_p}{\rho_t}\right)^{1/3}\left(\frac{mv^2}{Y}\right)^{1/3} \qquad (1)^{3\ 4}$$

$$p_\infty = 0\ 985 \times 10^{-3}\left(\frac{mv^2}{B}\right)^{1/3} \quad \text{(c g s units)} \qquad (2)^5$$

$$p_\infty = k_1(mv)^{1/3} \qquad (3)$$

$$p_\infty = 0\ 74\,\frac{\rho_p{}^{1/3}m^{1/3}}{\rho_t{}^{2/3}}\ln(1 + k_2v^2) \qquad (4)^7$$

$$p_\infty = k_3\left(\frac{m}{\rho_p}\right)^{1/3}\ln(1 + k_4v^2) \qquad (5)^8$$

where k_1, k_3, and k_4 are empirical constants,

$$k_2 = \frac{\rho_p{}^{2/3}\rho_t{}^{1/3}v^2}{4gB} \qquad (6)$$

ρ_p and ρ_t are the densities of projectile and target, respectively, m and v are the mass and velocity of the projectile, Y and B are, respectively, Young's modulus and Brinnell hardness number of the target, and g is the acceleration of gravity

The foregoing equations have been, in some cases, rewritten from their original forms to make the relationship between mass, velocity, and penetration clearer, since estimates of the masses and velocities of meteoroids are available from astronomical observations By these equations, penetration is related to kinetic energy, momentum, or to other more complicated functions of mass and velocity In this paper we shall assume, primarily for simplicity, that penetration is a function of kinetic energy as described by Eq (2) The method of calculating the penetrating flux which will be presented here is easily adaptable to the use of other functions describing penetration

The relationship between hypervelocity penetration of plates and the depth of penetration into a semi-infinite

Presented as Preprint 63-195 at the AIAA Summer Meeting, Los Angeles, Calif , June 17–20, 1963; revision received February 14, 1964 Gary Goodman of Northrop Space Laboratories has, by several informative discussions on probability, materially helped in the preparation of this paper

* Supervisor, Pyrotechnics Branch

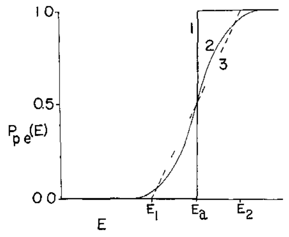

Fig. 1 Conditional probability of penetration

target is not very clear, but it is widely assumed that a hypervelocity projectile which produces a hemispherical crater of depth p_∞ in a semi-infinite target can penetrate plates up to a thickness given by

$$t = 1.5p_\infty \tag{7}$$

According to Ref. 9, the factor 1.5 results from penetration experiments up to 13,000 fps as reported in Ref. 10.

By choosing a single penetration equation, past investigators of the meteoroidal hazard have implicitly assumed that there are no penetrations up to some threshold E_a and that, above that threshold, all projectiles penetrate (curve 1 of Fig. 1). That threshold might be thought of as the energy-absorbing capability of the structure. Actually, the transition between zero penetrations and 100% penetrations occurs gradually (curve 2 of Fig. 1), and the point at which 50% of the projectiles penetrate is called the ballistic limit. In this figure, E represents energy, and p_p is the conditional probability of penetration, that is, the probability of penetration given a hit. Different curves of the probability of penetration will result depending on the physical characteristics of the material to be penetrated. Presumably for multiple-layer structures, curves similar to curve 2 of Fig. 1 could be obtained empirically.

If all incoming meteoroids had an equal energy E, and the total flux of meteoroids through a unit area in a unit time were F, then the penetrating flux would be given by

$$F_p = Fp_p\ (E) \tag{8}$$

where $p_p\ (E)$ is the conditional probability of penetration as a function of E. Since meteoroids vary widely in their kinetic energies, an integral process must be used to calculate F_p:

$$F_p = \int_{E_0}^{\infty} F(E)p_p\ (E)dE \tag{9}$$

In this integral, $F(E)$ represents the flux of meteoroids of exactly energy E. The lower limit of integration E_0 is chosen where $p_{p\,e}(E) = 0$. Before this integral can be evaluated, $F(E)$ must be obtained from available astronomical data, and $p_p\ (E)$ must be estimated from experimental work on hypervelocity impact. First $F(E)$ will be discussed and then $p_p\ (E)$.

Figure 2 gives the distribution of meteoric velocities V_∞ corrected to remove the accelerating effect of the earth's gravitational field. Both samples are based on meteors simultaneously photographed in New Mexico by two Baker Super-Schmidt cameras and include meteors brighter than approximately the fourth magnitude. Hawkins and Southworth[11] analyzed a randomly chosen sample of 359 meteors of which 285 were sporadics and the remainder shower meteors by the short-trail method described in Ref. 12. McCrosky and Posen[13] reduced 2529 meteors by a less accurate graphical method described in Ref. 14.

The distribution of geocentric velocities V_G uncorrected for the earth's gravitational field which were obtained by McCrosky and Posen[13] has been plotted in Fig. 3. McKinley[15] has obtained uncorrected geocentric velocities of 10,933 meteors, and a velocity distribution based on his observations is given in Fig. 3. McKinley estimates that his equipment could detect meteors down to approximately the ninth magnitude. Consequently, meteors observed by radar are considerably less massive than meteors observed visually or photographically. The curves of the distribution of V_∞ show a definite bimodal shape, with a primary peak at about 20 km/sec and secondary peak at about 65 km/sec. The photographic and radar distributions of V_G continue to show a bimodal shape, but there is disagreement as to the location of the peaks. McKinley[15] reports that when, on several occasions, the sensitivity of the radio equipment was depressed to reduce the detection rate by a factor of approximately ten, implying that the faintest meteors detectable were of the sixth or seventh magnitude, the velocity distributions so obtained were indistinguishable from velocity distributions obtained with the equipment operating at full sensitivity, corresponding to a lower detection limit of the ninth magnitude. Therefore, the disagreement between photographic and radar distributions of V_G probably results from a variation of either, or both, the luminous and ionizing efficiencies of meteors as a function of velocity rather than from a change in the shape of the velocity distribution as a function of mass, and the shape of the velocity distribution is independent of mass.

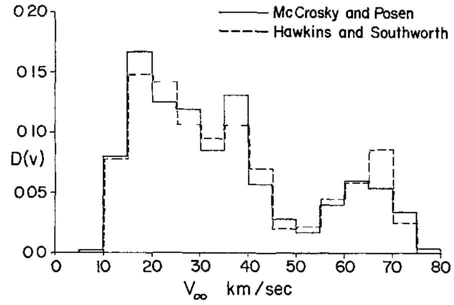

Fig. 2 Meteoric velocity distributions V_∞

Estimates of the masses of meteoroids are based on visual, photographic, and radar observations of meteors. Figure 4 gives the cumulative flux, i.e., the number of meteors of mass m, or greater, passing through a unit area per unit time, based on several sources. The cumulative flux can be represented by an equation of the form

$$\log F_>(m) = \log\alpha - \beta\log m \tag{10}$$

or

$$F_>(m) = \alpha m^{-\beta} \tag{11}$$

Estimates of α and β from different sources are given in Table 1.

The cumulative flux distribution may be expressed mathematically as the following integral in which $F(m)$ is the flux of meteoroids of mass, precisely m:

$$F_>(m) = \int_m^\infty F(m)dm \tag{12}$$

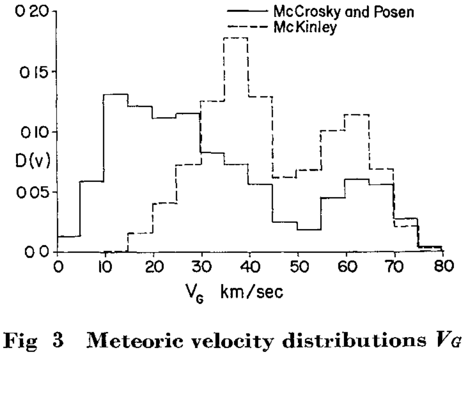

Fig. 3 Meteoric velocity distributions V_G

Solving the foregoing equation for $F(m)$ gives

$$F(m) = \alpha\beta m^{-\beta-1} \tag{13}$$

This is a density function of the cumulative flux distribution as a function of mass Similarly, the previously given velocity distributions may be thought of as approximations to a density function of a cumulative velocity distribution Let $\delta(v)$ be the velocity density function Then the flux density as a function of energy $F(E)$ is given by

$$F(E) = \int_{m_0}^{m} \delta(v)F(m)dm = \int_{2E/v_0^2}^{2E/v^2} \delta\left(\frac{2E}{m}\right)^{1/2} \alpha\beta m^{-\beta-1}\, dm \tag{14}$$

This equation assumes that the velocity distribution is not a function of mass Justification for this assumption has been given previously

Since the density function of the cumulative velocity distribution is available to us as a step function (Figs 2 and 3) rather than as a continuous function $\delta(v)$, the following summation can be used to find $F(E)$:

$$F(E)\Delta E = \sum_{i=0}^{n} D(v_i)\alpha\beta m_i^{-\beta-1}\Delta m_i \tag{15}$$

where $D(v_i)$ is the percentage of meteors between velocity limits centered on v_i, and Δm_i is the width of the ith interval Since

$$m_i = 2E/v_i^2 \tag{16}$$

$$\Delta m_i = 2\Delta E/v_i^2 \tag{17}$$

by substitution

$$F(E)\Delta E = \sum_{i=0}^{n} D_i \left[\frac{\alpha\beta v_i^{2(\beta+1)}}{(2E)^{\beta+1}}\right]\frac{2\Delta E}{v_i^2} = \frac{\alpha\beta\Delta E}{2^\beta E^{\beta+1}} \sum_{i=0}^{n} D_i v_i^{2\beta} \tag{18}$$

Let

$$S = \frac{1}{2^\beta}\sum_{i=0}^{n} D_i v_i^{2\beta} \tag{19}$$

Then

$$F(E) = \alpha\beta S E^{-\beta-1} \tag{20}$$

and this is the function needed in order to calculate the penetrating flux by means of Eq (9) The penetrating flux is proportional to the constant S, values of which are given in Table 2 as calculated using the velocity distributions of Figs 2 and 3 and different values of β

Now the integral in Eq (9) can be solved analytically provided that p_p (E), the conditional probability of penetration

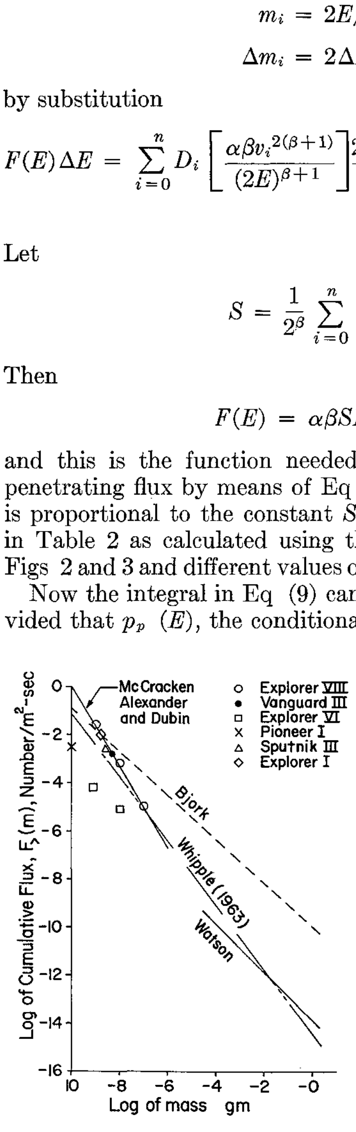

Fig 4 Cumulative meteoroidal flux

Table 1 Estimates of α and β

α $\frac{g}{m^2\text{-sec}}$	β	Source
1 06 $\times$ 10^{-14}	1 0	Watson[16]
1 0 $\times$ 10^{-12}	1 11	Bjork[6]
3 32 $\times$ 10^{-15}	1 34	Whipple[17]
1 0 $\times$ 10^{-17}	1 70	McCracken, Alexander, and Dubin[18]

as a function of energy, is well behaved and zero below energy levels where solar radiation pressure causes the cumulative flux curve $F_>(m)$ to become nonlinear If p_p (E) has the form of curve 3 of Fig 1, by substitution into Eq (9),

$$F_p = \alpha S\left[\frac{\beta(E_2^{-\beta+1} - E_1^{-\beta+1})}{(E_2 - E_1)(-\beta+1)} + \frac{E_1(E_2^{-\beta} - E_1^{-\beta})}{E_2 - E_1} + E_2^{-\beta}\right] \qquad \beta \neq 1$$

$$= \frac{\alpha S}{E_2 - E_1}[\log E_2 - \log E_1] \qquad \beta = 1 \tag{21}$$

As curve 3 is made steeper and steeper, the penetrating flux approaches, as a limit, the penetrating flux calculated using curve 1 of Fig 1:

$$F_p = \alpha S E_a^{-\beta} \tag{22}$$

Now we can compare the penetrating flux calculated assuming that p_p (E) is a step function [Eq (22) and curve 1 of Fig 1] with the penetrating flux calculated assuming a more gradual variation of p_p (E) [Eq (21) and curve 3 of Fig 1] By Ref 19, the spread from zero probability of penetration to 100% probability of penetration might be on the order of $\pm 3\%$, with the spread entirely attributable to variations in velocity measurement and in the properties of the projectile and the target By setting

$$E_2 = E\,(1+\epsilon) \tag{23}$$

$$E_1 = E_a(1-\epsilon) \tag{24}$$

and substituting into Eq (21) with $\beta = 1$, we find

$$F_p \doteq (\alpha S/E\ )[4/(4-\epsilon^2)] \tag{25}$$

For the 3% spread just suggested (ϵ = 0 03), the error introduced into the penetrating flux calculated under the assumption that p_p (E) is a step function rather than a gradually varying function is negligible, approximately 0 02%

It may be of interest to use our more rigorous integral process to check a recent estimate of the penetrating flux Reference 17 has calculated the penetrating flux on the following assumptions: 1) a step change in the probability of penetration (curve 1 of Fig 1); 2) the Herrmann and Jones penetration equation [Eq (4)]; 3) a factor of 1 5 to convert from penetration of a semi-infinite target into penetration of a thin target [Eq (7)]; 4) a meteoroidal density of 0 44 g/cm³; 5) a meteoroidal velocity of 22 km/sec; and 6) a cumulative meteoroidal flux as given by Eq (11), with α = 3 32 $\times$ 10^{-15} g^β/m^2-sec, and β = 1 34

For a certain thickness to be penetrated, there exists a constant C such that

$$C = \frac{t^3\rho_t^2}{(1\ 5)^3(0\ 74)^3\rho_p} = m[\ln(1 + k_2v^2)] \tag{26}$$

Then, by substitution into Eq (11), the penetrating flux, as calculated in Ref 17 for an assumed average velocity v_{av}, may be written parametrically as

$$F_p = \alpha W C^{-\beta} \tag{27}$$

where

$$W = [\ln(1 + k_2v^2)]^{3\beta} \tag{28}$$

The following calculation of the penetrating flux follows all of the assumptions of Ref 17 except for the assumption of a constant impact velocity In place of that assumption, we shall assume that meteoroids have a velocity distribution that is independent of mass Then, following a procedure analogous to that of Eqs (14–20),

$$F(C) = \int_{m_n}^{m_0} \delta(v)F(m)dm \tag{29}$$

where

$$F(m) = \alpha\beta m^{-\beta-1} \tag{30}$$

$$m_n = \frac{C}{[\ln(1 + k_2v_n^2)]^3} \tag{31}$$

$$m_0 = \frac{C}{[\ln(1 + k_2v_0^2)]^3} \tag{32}$$

Changing, as before, to a summation

$$F(C)\Delta C = \sum_{i=0}^{n} D(v_i)\alpha\beta m_i^{-\beta-1}\Delta m_i \tag{33}$$

Since

$$m_i = \frac{C}{[\ln(1 + k_2v_i^2)]^3} \tag{34}$$

$$\Delta m_i = \frac{\Delta C}{[\ln(1 + k_2v_i^2)]^3} \tag{35}$$

by substitution

$$F(C)\Delta C = \sum_{i=0}^{n} D_i \frac{\alpha\beta[\ln(1 + k_2v_i^2)]^{3(\beta+1)}}{C^{\beta+1}} \left\{\frac{\Delta C}{[\ln(1 + k_2v_i^2)]^3}\right\} \tag{36}$$

$$F(C) = \frac{\alpha\beta}{C^{\beta+1}} \sum_{i=0}^{n} D_i[\ln(1 + k_2v_i^2)]^{3\beta} \tag{37}$$

By analogy with Eq (9), the penetrating flux, using our rigorous integral method, is given by

$$F_p = \int_{C_0}^{\infty} F(C)p_p\ (C)dC \tag{38}$$

We have previously shown that assuming an abrupt change the conditional probability of penetration introduces a negligible error Therefore, assuming that p_p (C) changes from 0 to 1 at $C_0 = C$,

$$F_p = \int_{C}^{\infty} F(C)dC \tag{39}$$

By substituting Eq (37) into Eq (39) and integrating

$$F_p = \alpha S^* C^{-\beta} \tag{40}$$

where

$$S^* = \sum_{i=0}^{n} D_i[\ln(1 + k_2v_i)^2]^{3\beta} \tag{41}$$

For a spaceship with a skin of 5056 aluminum, which has a density of 2 64 g/cm^3 and a Brinnell hardness number of 105, by substitution into Eq (41), using the distribution of V_∞ of McCrosky and Posen,[13]

$$S^* = 921 \tag{42}$$

If McKinley's[15] distribution of V_G as obtained by radio is used, then

$$S^* = 1288 \tag{43}$$

When the same values of density and Brinnell hardness number and v_v = 22 km/sec are substituted into Eq (28), we find

$$W = 443 \tag{44}$$

Accordingly, Ref 17 underestimates the penetrating flux by a factor of about two or three, depending on which velocity distribution is used The sensitivity of the penetrating flux to changes in the velocity distribution is obvious, and this suggests that the orbit of a space vehicle, since it affects the velocity distribution of meteoroids encountered, should be considered in calculating the penetrating flux The rate of encountering meteoroids is also affected by the orbit, and it can be shown that a more rapidly moving satellite will encounter more meteoroids than a slowly moving satellite, other things being equal The geometry of the body exposed to meteoroids and the shielding effect of the earth will also affect the penetrating flux We have not considered how the penetrating flux is affected by the variation of the probability of penetration as a function of angle, primarily because of a lack of experimental data This effect could probably be included by modifying Eq (9) to include integration with respect to impact angle Mar[20] has already made a step in this direction by using integral calculus to calculate a mean effective skin thickness different from the actual skin thickness of the satellite

Once the penetrating flux has been calculated, the probability of penetration can be found In the past, the Poisson distribution has been universally used to calculate $p(n)$, the probability of a certain number of penetrations, after some estimate of the penetrating flux has been made The probability of exactly zero penetrations through an area A during a time t is given by the Poisson distribution as

$$p(0) = \frac{(F_pAt)^0}{0!} e^{-F_pAt} \tag{45}$$

$$p(0) \doteq 1 - F_pAt \tag{46}$$

when F_pAt is small

Gallagher and Eshelman[21] remark that their radar observations of the meteoric flux rate are better fitted by a normal distribution than by a Poisson distribution Davison and Winslow,[22] analyzing Olivier's[23] visual flux measurements, state that the observations in the latter reference are not very well fitted by any single theoretical distribution Reference 22 contains some plots of data showing some groups of observations taken from Ref 23 which are better fitted by a normal or log normal distribution than by a Poisson distribution

The probability of zero penetrations, using a normal distribution, is

$$p(0) = \frac{1}{(2\pi)^{1/2}\sigma_n} \int_{-1/2}^{1/2} \exp \frac{-(n - N)^2}{2\sigma_n} dn \tag{47}$$

Table 2 Estimates of the constant S

$D(v)$	S, (km/sec)$^{2\beta}$ $\beta = 1\ 0$	$\beta = 1\ 11$	$\beta = 1\ 34$	$\beta = 1\ 70$
Hawkins and Southworth (V_∞)	796	1754	8940	128,400
McCrosky and Posen (V_∞)	766	1670	8640	120,700
McCrosky and Posen (V_G)	697	1523	7937	111,300
McKinley (V_G)	1100	2449	12 850	178,300

Gallagher and Eshelman found that the normal curve that best fits their data has a ratio of the expected number N to the standard deviation σ_n of 3 36 If other observations followed a normal distribution of this dispersion, the zero probability would be different from that predicted by the Poisson distribution, the extent of the difference being shown in Fig 5 where the curve labeled "normal" has been calculated from the foregoing probability integral assuming $N/\sigma_n = 3\ 36$ The dangers of an indiscriminate assumption that the Poisson distribution is applicable are apparent

Summary and a View to the Future

We have demonstrated a rigorous process using integral calculus to calculate the penetrating flux of meteoroids, and have shown that the gradual transition fiom zero probability of penetration to 100% probability of penetration has a negligible effect on the estimate of the penetrating flux

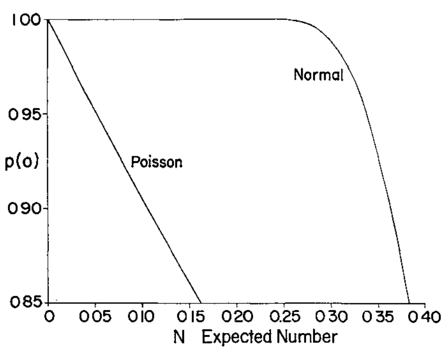

Fig 5 Probability of no penetration

The penetrating flux so calculated is dependent on the measuied velocity distribution of meteors which is in turn strongly dependent on either the luminous efficiency or the ionizing efficiency of meteors, according to whether the velocity dis tribution is obtained by optical-visual observations or by radar observations Urgently needed are more accurate values of luminous and ionizing efficiencies as a function of velocity Also needed are more accurate measurements of the flux constants α and β and further investigations of the phenomena of penetration higher in the hypervelocity region and at oblique impact angles A rigorous mathematical analysis of the effect of the orbit of a spaceship on the rate of encountering meteoroids and on the velocity distribution of those encountered remains to be done We have discussed biiefly the use of distributions other than the Poisson to describe the arrival of meteoroids A more thorough analysis of the statistics involved is needed, particularly with regard to the variation of the strength of meteor radiants with celestial latitude and longitude and with time Although much work remains to be done, theoretical analyses, astronomical and satellite observations, and hypervelocity-impact experiments are rapidly improving our ability to predict the probability of meteoroidal penetration

References

[1] Rinehart, J S and Pearson, J, *Behavior of Metals under Impulsive Loads* (American Society for Metals, Cleveland, Ohio, 1954), pp 124–134

[2] Gell, C F, Thompson, A B, and Stembridge, V, "Biological effects of simulated micrometeoroid penetration of a sealed chamber containing animal specimens" Aerospace Medicine **33,** 156–161 (1962)

[3] Charters, A C and Locke, G S, "A preliminary investigation of high speed impact, the penetration of small spheres into thick copper targets, ' NASA RM A58B26 (1958)

[4] Summers, J L, "Investigation of high-speed impact: regions of impact and impact at oblique angles, ' NASA TN D 94 (October 1959)

[5] Eichelberger, R J and Gehring, J W, "Effects of meteoroid impacts on space vehicles ' ARS J **32,** 1583–1591 (1962)

[6] Bjork R L, ' Meteoroids vs space vehicles," ARS J **31,** 803–807 (1961)

[7] Herrmann, W and Jones, A H, ' Correlation of hypervelocity impact data,' *Proceedings of the Fifth Symposium on Hypervelocity Impact* (Colorado School of Mines, Denver Colo, 1962), Vol 1, Part 2 pp 389–438

[8] Yuan, S W and Scully, C N, 'New approach to the hypervelocity impact theory," Ninth Meeting of the American Astronautical Society, Los Angeles, Calif (January 15–17, 1963)

[9] Nysmith, C R and Summers, J L, Preliminary investigation of impact on multiple-sheet structures and an evaluation of the meteoroid hazard to space vehicles" NASA TN D 1039 (September 1961)

[10] Kinard, W H, Lambert, C H, Schryer, D R, and Casey, F W, ' Effect of target thickness on cratering and penetration of projectiles impacting at velocities to 13 000 feet per second,' NASA Memo 10 18-58L (1958)

[11] Hawkins G S and Southworth, R B, ' The statistics of meteors in the earth's atmosphere," Smithsonian Contrib Astrophys **2,** no 11, 349–364 (1958)

[12] Hawkins, G S, ' The method of reduction of short trail meteors, Smithsonian Contrib Astrophys **1,** no 2, 207–214 (1957)

[13] McCrosky, R E and Posen A, ' Orbital elements of photographic meteors ' Smithsonian Contrib Astrophys **4,** no 2, 15–84 (1961)

[14] McCrosky, R E, ' A rapid method of meteor trail reduction," Smithsonian Contrib Astrophys **1,** no 2, 215–224 (1957)

[15] McKinley, D W R, ' Meteor velocities determined by radio observations," Astrophys J **113,** 225–267 (March 1951)

[16] Watson, F G, *Between the Planets* (Blakiston Co, Philadelphia, Pa, 1941), pp 114–117

[17] Whipple, F L, "On meteoroids and penetration, ' J Astionaut Sci **X,** no 3, 92–94 (1963)

[18] McCracken, C W, Alexander, W M, and Dubin, M, "Direct measurement of interplanetary dust particles in the vicinity of the earth,' Nature **192,** 441–442 (November 1961)

[19] Summers, J L, private communication (March 1963)

[20] Mar, J, "Meteoroid impact on the topside sounder satellite," Canadian Aeronaut Space J **8,** 237–240 (November 1962)

[21] Gallagher, P B and Eshelman, V R, " 'Sporadic shower properties of very small meteors ' J Geophys Res **65,** 1846–1847 (1960)

[22] Davison, E H and Winslow P C, "Direct evaluation of the meteoroid hazard ' Aerospace Eng **21,** 24–33 (February 1962)

[23] Olivier, C P, "Catalog of hourly meteor rates,' Smithsonian Contrib Astrophys **4,** no 1, 1–14 (1960)